Competing Matchmaking

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Abstract: We study how matchmakers use prices to sort heterogeneous participants into competing matching markets, and how equilibrium outcomes compare with monopoly in terms of prices, matching market structure and sorting efficiency under the assumption of complementarity in the match value function. The role of prices to facilitate sorting is compromised by the need to survive price competition. We show that price competition leads to a high quality market that is insufficiently exclusive. As a result, the duopolistic outcome can be less efficient in sorting than the monopoly outcome in terms of total match value in spite of servicing more participants.

Keywords: Overtaking, complementarity, market structure, market coverage, market differentiation

JEL codes: C7, D4

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1. Introduction

Since the seminal work on network competition by Katz and Shapiro (1985) (see also Farrell and Saloner, 1986; Fujita, 1988), there has been a growing economic literature on competing marketplaces. This literature reflects the importance of network externalities in industries ranging from telecommunications to software platforms and to credit cards. In these industries, a competing marketplace is a network (platform) on which participants interact, and agents' network choices have external effect on each other's welfare. In most of the earlier works of the literature, the driving force is the thick market effect that a larger network provides a greater chance of finding a trading partner. This positive size effect favors the dominance of a single marketplace, and a central question is whether and when multiple marketplaces can coexist in equilibrium.[1] Although the size effects are important in network competition, in many industries network participants also care about the identities of other participants in the same network. For example, in markets such as job search, real estate and dating, where networks are intermediaries, participants are heterogeneous and networks differ not only in relative size but also in quality. In these markets participants' network choices can have external effect on each other's welfare by changing the composition and hence the quality of the network pool. This type of "sorting externality" and its implications to price competition have been neglected in the literature on competing marketplaces, which focuses on the size effects and assumes either that agents are homogeneous or that agents' choice of network is independent of their type.[2]

This paper introduces a model of price competition among marketplaces in an environment where agents have heterogeneous qualities, and where the expected quality of the

[1] More recently, research in this literature has increasingly focused on two-sided marketplaces, where participants are interested in matching with those on the other side. Ellison and Fudenberg (2003) and Ellison, Fudenberg and Mobius (2004) reexamine the coexistence of multiple networks by allowing a negative size effect that the agents to prefer networks with fewer competitors, Caillaud and Jullien (2001, 2003) and Rochet and Tirole (2003) analyze the "divide-and-conquer" strategy of subsidizing one side of the market while recovering the loss from the other side, and Armstrong (2004) studies the implications on price competition of "multi-homing" where participants on the two sides can choose multiple networks.

[2] Ambrus and Argenziano (2004) modify the framework of Caillaud and Jullien (2001; 2003) and allow for heterogeneous preferences. Agents have the same quality but differ in terms of willingness to pay for participating in a larger network. In their model, the equilibrium distribution of participant types can be different across networks, however the size effect remains the only externality.

pool of participants affects agents' decision of which marketplace to join. In our model, a marketplace is a random matching market, or more specifically, a meeting place where participants randomly match with each other. We have in mind a job market or a dating market, where agents have private information about their one-dimensional quality characteristics (type), and where the match value function exhibits complementarity between types. Since type information is private, agents self-select into matching markets based on the prices and their expectations of the quality of the pool in the matching market. Under the assumption of complementarity, how agents sort into the matching markets by type has implications to efficiency in terms of total match value. The random matching technology we adopt implies the absence of any size effect. This allows us to isolate the implications of the sorting externality introduced in this paper from the consequences of the much studied size effects. We stress that our analytical framework applies equally well to one-sided intermediary markets, such as private schools that compete with tuition charges and country clubs that compete with membership fees. In these applications, instead of random pairwise match formation in each matching market (such as a school or a club), we can allow any form of interaction among the participants so long as the reduced form payoff function exhibits complementarity between the individual type and the average type. Thus the present paper, by introducing price competition in an oligopoly model, also contributes to the literature on locational choices where the peer effect plays a critical role (De Bartolome, 1990; Epple and Romano, 1998).[3]

In section 2 we lay out the framework of duopoly price competition in a matching environment. In our model, matchmakers use prices (subscription fees) to induce agents to sort into different matching markets. We introduce the concept of matching market structure, which describes how agents sort into two matching markets given the two prices. We then provide a criterion to select a unique market structure for any price profile. Price competition in a matching environment with friction differs from the standard Bertrand models because prices also play the role of sorting heterogeneous agent types into different

[3] As a model of one-sided intermediated market, our paper is also related to the literature on demand externalities and pricing (Karni and Levin, 1994; Rayo, forthcoming), and to the literature on clubs (Cole and Prescott, 1997).

matching markets. Aside from the usual strategy of lowering price to steal rivals' market share, our selection criterion formalizes a pricing strategy called "overtaking" that is unique to the sorting role of prices. Overtaking a rival is achieved by charging a price just higher than the rival does, and thus providing a market with a higher quality (average agent type). When the price difference is small enough, the rival's matching market loses all its customers because quality difference dominates.

Section 3 contains the main results of the paper. No pure-strategy equilibrium exists in the simultaneous-move pricing game, because for any price profile at least one of the matchmaker has an incentive to drive its rival out of the market by using the overtaking strategy. We provide a sufficient condition for the two matchmakers to coexist in the equilibrium of the sequential-move version of the pricing game. This condition requires the type distribution to be sufficiently diffused so that the first mover can create a niche market for the low types to survive the overtaking strategy of the second mover, which in equilibrium serves the higher types. With the assumption of uniform type distribution, we show that at the equilibrium outcome of the duopoly competition the total market coverage is greater than the optimal total coverage under a monopolist that maximizes revenue from two matching markets, because the first mover must lower its price to prevent overtaking. However, the equilibrium outcome involves inefficient sorting compared to the monopoly outcome, because competition results in an insufficiently exclusive high quality matching market. When the type distribution is tight, the matching market structure is less efficient overall under competition than under monopoly, as the loss from inefficient sorting outweighs the gain from a greater coverage. We conclude our analysis with a brief discussion of the robustness of our main results when the type distribution is non uniform, and when more than two matching markets are created. Section 4 provides further remarks on the existing literature and the implications of our results to regulatory policies in intermediated markets. Proofs of all lemmas can be found in the appendix.

2. A Duopoly Model of Competing Matchmakers

Consider a two-sided matching environment. Agents of the two sides have heterogeneous one-dimensional characteristics, called "types." For simplicity, we assume that the two sides have the same size and the same type distribution function F, with a support $[a, b] \subseteq \mathbb{R}_+$, and a differentiable density function f. We assume that $a > 0$ and b is finite; the following analysis carries through with appropriate modifications if $a = 0$ or $b = \infty$, and all our results hold without change.

Two matchmakers, unable to observe types of agents, use prices (entrance fees) to create two matching markets.[4] For each $i = 1, 2$, let p_i be the price charged by matchmaker i. Given p_1 and p_2, agents simultaneously choose one from three options: participate in matchmaker 1's matching market, participate in 2's market, and not participate. In each matching market, agents are randomly pairwise matched. Random matching means that the probability that a type x agent meets an agent from the other side whose type is in some set equals the proportion of matching market participants whose type belongs to the set. We assume that matching markets are costless to organize, and each matchmaker's objective is to maximize the sum of entrance fees collected from participants.[5]

A match between a type x agent and a type y agent from the other side produces a value of xy to both of them. This match value function satisfies the standard complementarity condition (positive cross partial derivatives); this implies that in a frictionless matching environment, the total match value is maximized by matching equal types of agents. Let m_i, $i = 1, 2$, be the expected type (average quality) in the matching market created by matchmaker i; the qualities m_1 and m_2 are endogenously determined in equilibrium by p_1 and p_2, and the participation choices of the agents. The utility of a type x agent from participating in matching market i is then $xm_i - p_i$. Unmatched agents get a payoff of 0 regardless of type.

2.1. Matching market structures

First we examine the Nash equilibria of the simultaneous move game played by the agents for given prices p_1 and p_2. For concreteness, we refer to each equilibrium as a "matching

[4] Due to the assumptions of symmetry and random pairwise meeting, in our model each participant in a matching market is matched with probability 1. If the meeting technology is such that the probability of forming a match is less than 1, then the prices should be understood as usage fees that are paid only if a successful match is made, and all analysis remains unchanged.

[5] The same framework can be used to analyze the optimal pricing of a single matchmaker that competes with a free-access matching market. See our earlier paper, Damiano and Li (forthcoming).

market structure." Since our model is symmetric with respect to the two sides, we restrict our attention to symmetric Nash equilibria, with each matching market hosting an equal number of participants with identical support from the two sides. For any $c, c' \in [a, b]$ with $c < c'$, let $\mu(c, c')$ be the mean type on the interval $[c, c']$, and denote $\mu(c, c) = c$.

One "singular matching market structure," denoted as S_1, is that agents participate in matching market 1 only. The participation threshold c_1 for matching market 1 is determined by

$$\begin{cases} c_1\mu(c_1, b) = p_1 & \text{if} \quad p_1 \in [a\mu(a, b), b^2]; \\ c_1 = a & \text{if} \quad p_1 \in [0, a\mu(a, b)). \end{cases} \tag{2.1}$$

The average quality m_1 of matching market 1 is $\mu(c_1, b)$, and the threshold participation type is either a type c_1, which is indifferent between participating in matching market 1 and not participating (when $p_1 \geq a\mu(a, b)$), or the lowest type a, which strictly prefers participation (when $p_1 < a\mu(a, b)$). In both cases, all types higher than the threshold type strictly prefer to participate in matching market 1. The other singular matching market structure, denoted as S_2, is that agents participate in matching market 2 only; the average quality m_2 and the participation threshold c_2 are similarly determined.

When $p_1 < p_2 \leq b^2$, the prices may also support a "dual matching market structure," denoted as D_{12}. Either there exist participation thresholds c_1 and c_2, with $a \leq c_1 < c_2$, such that

$$\begin{aligned} &c_1\mu(c_1, c_2) = p_1; \\ &c_2(\mu(c_2, b) - \mu(c_1, c_2)) = p_2 - p_1, \end{aligned} \tag{2.2}$$

or there is c_2 such that

$$\begin{aligned} &a\mu(a, c_2) > p_1; \\ &c_2(\mu(c_2, b) - \mu(a, c_2)) = p_2 - p_1. \end{aligned} \tag{2.3}$$

In both cases above, the average quality of matching market 2 is $m_2 = \mu(c_2, b)$, and the threshold type c_2 is indifferent between the two markets. In the first case, the threshold type c_1 is indifferent between participating in matching market 1 with the average quality $m_1 = \mu(c_1, c_2)$ and not participating at all, while in the second case, type c_1 is the lowest type a and it strictly prefers participating in matching market 1 with $m_1 = \mu(a, c_2)$. Whether a pair of prices p_1 and p_2 with $p_2 < p_1$ supports a symmetric dual matching market structure, denoted as D_{21}, is determined similarly.

The assumption of complementarity in the match value function implies that participation decisions can be described by thresholds, and in any dual matching market structure higher types join the more expensive market. As a result, the two singular matching market structures and the dual structure, together with the "null matching market structure" where agents participate in neither matching market, cover all possible equilibrium matching market structures.[6]

We now make an assumption which for each $p_1 \in [0, b^2]$ allows to determine a price range $[\theta(p_1), \lambda(p_1)]$ for prices $p_2 > p_1$, such that the dual matching market structure D_{12} cannot be supported for any $p_2 < \theta(p_1)$ or $p_2 > \lambda(p_1)$ and there is a unique D_{12} for any $p_2 \in [\theta(p_1), \lambda(p_1)]$. The lower bound $\theta(p_1)$ is given by

$$\theta(p_1) = \begin{cases} p_1 + \sqrt{p_1}(\mu(\sqrt{p_1}, b) - \sqrt{p_1}) & \text{if} \quad p_1 \geq a^2; \\ p_1 + a(\mu(a, b) - a) & \text{if} \quad p_1 < a^2 \end{cases} \tag{2.4}$$

and the upper bound $\lambda(p_1)$ is given by

$$\lambda(p_1) = \begin{cases} p_1 + b(b - \mu(c_1, b)) & \text{if} \quad p_1 \geq a\mu(a, b); \\ p_1 + b(b - \mu(a, b)) & \text{if} \quad p_1 < a\mu(a, b), \end{cases} \tag{2.5}$$

where c_1 is uniquely determined by $c_1\mu(c_1, b) = p_1$ in the first case of (2.5). The assumption we will make (Assumption 2.1 below) implies that $\mu(t, x') - \mu(x, t)$ is a non-decreasing function in t for any $t \in (x, x') \subset [a, b]$. To see the sufficiency of this monotonicity condition on the difference of conditional means, take for example the case of $p_1 \geq a\mu(a, b)$. At $p_2 = \theta(p_1)$, equation (2.2) is satisfied by $c_1 = c_2 = \sqrt{p_1}$; under the monotonicity condition, as p_2 decreases c_2 decreases while c_1 increases, and thus there is no solution in c_1 and c_2 with $c_1 < c_2$ if $p_2 < \theta(p_1)$. Similarly, at $p_2 = \lambda(p_1)$, equation (2.2) is satisfied by $c_2 = b$ and c_1 such that $c_1\mu(c_1, b) = p_1$; under the monotonicity condition, there is no solution in c_1 and c_2 to equations (2.2) if $p_2 > \lambda(p_1)$. Lastly, for $p_2 \in [\theta(p_1), \lambda(p_1)]$, the monotonicity condition implies that there is a unique pair of participation thresholds c_1 and c_2 that satisfies equation (2.2). The cases of $p_1 < a^2$ and $p_1 \in [a^2, a\mu(a, b))$ can be similarly established. A symmetric argument holds for D_{21}.

[6] When $p_1 = p_2$, for the participation threshold c that satisfies (2.1), any strategy profile such that types above c join one of the two markets and $m_1 = m_2$ forms a Nash equilibrium. We assume that the two matchmakers evenly split the types above c; the analysis is unaffected by this assumption.

To derive a restriction on F that ensures that $\mu(t,x') - \mu(x,t)$ is non-decreasing in t, let μ_l be the partial derivative of $\mu(x,x')$ with respect to x, and μ_r be the derivative with respect to x'. We have

$$\mu_l(t,x') = \frac{f(t)(\mu(t,x') - t)}{F(x') - F(t)}; \quad \mu_r(x,t) = \frac{f(t)(t - \mu(x,t))}{F(t) - F(x)}. \tag{2.6}$$

Note that $\mu_l(t,x')$ converges to $\frac{1}{2}$ as x' approaches t.[7] Further, the derivative of $\mu_l(t,x')$ with respect to x' has the same sign as $\frac{1}{2}(t+x') - \mu(t,x')$, which is non-negative if $f'(\cdot) \leq 0$. Thus, $\mu_l(t,x') \geq \frac{1}{2}$ if $f'(\cdot) \leq 0$. Similarly, $\mu_r(x,t)$ converges to $\frac{1}{2}$ as x approaches t, and is non-decreasing in x if $f'(\cdot) \leq 0$, implying that $\mu_r(x,t) \leq \frac{1}{2}$. Non-increasing density is thus sufficient to imply that $\mu(t,x') - \mu(x,t)$ is non-decreasing in t as $\mu_l(t,x') \geq \frac{1}{2} \geq \mu_r(x,t)$. We make the following assumption.

Assumption 2.1. *The density function f is non-increasing.*

For the analysis that we will carry out, we also need the standard assumption of monotone hazard rate. Let $\rho(\cdot) = (1 - F(\cdot))/f(\cdot)$ be the inverse hazard rate function. We assume that $\rho'(\cdot) \leq 0$. This is equivalent to the assumption that the right tail distribution function $1 - F(\cdot)$ is log-concave, which implies that the conditional mean function $\mu(t,b)$ satisfies $\mu_l(t,b) \leq 1$ (An, 1998).

Assumption 2.2. *The hazard rate function of F is non-decreasing.*

The uniform distribution and the exponential distribution are the two polar cases that satisfy Assumptions 2.1 and 2.2. The uniform distribution on $[a,b]$ has a constant density, while the hazard rate is strictly increasing. The exponential distribution on $[a,\infty)$ has a strictly decreasing density, while the hazard rate is constant.

2.2. Selection of matching market structures

Unlike in standard Bertrand price competition, in a matching environment participation decisions of agents are not completely determined by prices. What an entrance fee buys

[7] The derivative $\partial\mu(t,x')/\partial t$ at $x' = t$ can be calculated using L'Hospital rule and solving for it from the resulting equation. It is equal to $\frac{1}{2}$ because a continuous density is locally uniform.

for agents on one side of the matching market depends on participation decisions by the agents on the other side of the market. Nash equilibrium alone does not pin down the matching market structure. It is possible to have multiple matching market structures for a given pair of prices. Indeed, from equations (2.1), for any $p_1, p_2 \in [0, b^2]$, both the two singular matching market structures S_1 and S_2 can be supported as equilibrium.

We adopt as our selection criterion the notion of "stable set of equilibria" of Kohlberg and Mertens (1986). Their notion is a strengthening of trembling hand perfection in strategic-form games (Selten, 1975), and is derived from robustness considerations in perturbed games where agents are constrained to non-optimal participation decisions (trembles) with increasingly small probabilities. Loosely speaking, in our model a collection of matching market structures constitutes a stable set in the sense of Kohlberg and Mertens if it is a minimal collection with the property that every perturbed game has a Nash equilibrium close to some matching market structure in the collection. In the appendix, we give a formal definition of the notion of a stable collection of matching market structures and prove the following result.[8]

LEMMA 2.3. *Assume $p_1 < p_2$. The unique stable collection of matching market structures is a singleton, and contains (i) S_2 if $p_2 < \theta(p_1)$, (ii) D_{12} if $\theta(p_1) < p_2 < \lambda(p_1)$, and (iii) S_1 if $p_2 > \lambda(p_1)$.*

Stability in the sense of Kohlberg and Mertens makes a unique selection of matching market structure, even though the concept is a set-based refinement as in general different equilibria are needed to provide robustness against different perturbed games. To understand this strong result, let us consider case (i) where $p_2 \in (p_1, \theta(p_1))$. In this case, the price difference is too small to support the dual matching market structure D_{12}, and the unique selection is the high-price singular market structure S_2. The low-price singular structure S_1 is not robust. This is because any perturbation in which high types are over-represented in the high-price market would create a high quality there, and since the price difference is small, would further attract high types. As high types leave the low-price

[8] At the boundary between S_2 and D_{12} (when $p_2 = \theta(p_1)$), the two matching market structures are both stable, but since they are outcome-equivalent, which one is selected is immaterial to our analysis. A similar observation applies to the boundary between D_{12} and S_1 (when $p_2 = \lambda(p_1)$).

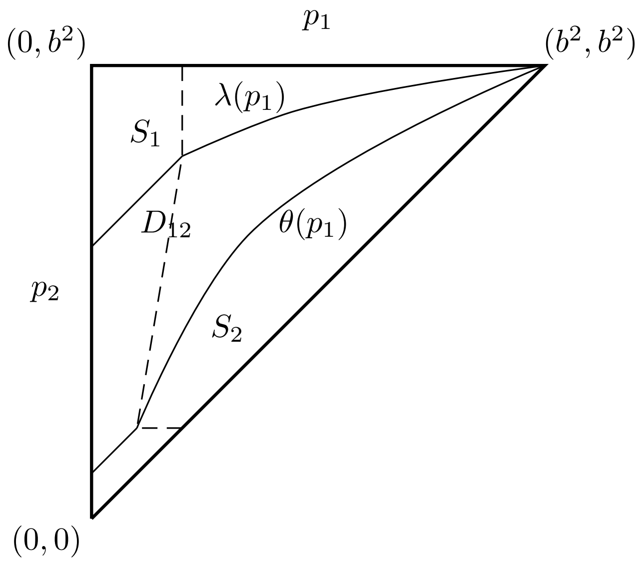

Figure 1

market, its quality decreases. This induces further deviations that unravel the low-price singular market structure.[9] In contrast, the high-price structure S_2 is robust. In any perturbation, the first types to deviate to the low-price market are the low types, which drives up the quality difference between the two markets and limits further deviations.

By a symmetric argument a unique matching market structure is selected when $p_1 > p_2$. Figure 1 depicts the selected matching market structure for the case in which types are uniformly distributed on $[a, b]$. The dashed line represents the border between the region with high prices and full participation, with $c_1 = a$, and low prices and partial participation, with $c_1 > a$. We refer to case (i) in Lemma 2.3 as matchmaker 2 "overtaking" matchmaker 1, and case (iii) as matchmaker 1 "undercutting" matchmaker 2. The strategy of overtaking is unique to the sorting role of prices. Overtaking a rival is achieved by charging an appropriately higher price than the rival does. This provides a higher quality market, inducing deviation from the rival's market by the highest types, which triggers further deviations by lower type agents and eventually drives out the rival. The overtaking strategy plays on the differences in willingness to pay for quality (average match type) between the highest and the lowest type agents participating in a market.

9 A similar argument can be made if we adapt the concept of most likely deviating type from Banks and Sobel's (1987) theory of refinement in extensive games. In the low-price singular structure S_1, the type that is the most likely to deviate to the high-price market is the highest type b. If $p_2 < \theta(p_1)$, type b agents would indeed want to deviate if they expect a sufficiently high quality in the high-price market.

3. Duopolistic Sorting

In this section we analyze the equilibrium outcome under duopolistic competition. First we have that no pure-strategy equilibrium exists in the simultaneous-move pricing game, because each matchmaker can drive the rival out of the market by overtaking.

Lemma 3.1. *There is no pure-strategy equilibrium in a simultaneous-move game.*

The non-existence of pure-strategy equilibria in the simultaneous-move game points to a difference between competing matchmaking and the standard Bertrand price competition. As in Bertrand competition, payoff discontinuities exist in competing matchmaking because the matching market structure switches from one singular matching market structure to the other singular market structure when prices move from p_1 just below p_2 to p_1 just above p_2. Payoff discontinuities tend to homogenize prices in the absence of any asymmetry between the competitors. While in Bertrand competition this leads to marginal cost pricing, the same is not true in competing matchmaking because prices also play the role of sorting. If one matchmaker charges zero price, the other matchmaker can charge a price in the region of dual market structure and earn a strictly positive revenue by sorting out the types willing to pay more for a higher match quality. Rather than studying mixed-strategy equilibria in a simultaneous-move game, we look at pure-strategy (subgame perfect) equilibria in a sequential-move game.[10] We consider below a game where matchmaker 1 first picks a price p_1, and matchmaker 2 then chooses p_2 after observing p_1.

3.1. Surviving overtaking

Because of the overtaking strategy, matchmaker 2 has an advantage in the sequential-move game. We want to know whether this advantage is so overwhelming that matchmaker 1 cannot survive as a first mover. A possible strategy for matchmaker 1 to survive overtaking

[10] Existence of a mixed-strategy equilibrium can be established using the concept of payoff-security of Reny (1999). By charging a slightly higher price each matchmaker can secure a payoff at worst only marginally lower against small perturbations of its rival's price. It follows that the mixed extension of our simultaneous-move game is payoff-secure, and therefore a mixed strategy equilibrium in prices exists (see Corollary 5.2 in Reny, 1999).

is to choose a price so low that matchmaker 2 finds more profitable creating a more exclusive matching market rather than overtaking matchmaker 1 and driving it out of the competition. We say that the type distribution is "sufficiently diffused" if $\mu(a,b) > \frac{3}{2}a$. Intuitively, when the type distribution is sufficiently diffused, there is room for two matchmakers to coexist, because the lowest type's willingness to pay for a higher quality match is low relative to higher type agents. When matchmaker 1 posts a sufficiently low price, overtaking effectively entails serving the entire market. The opportunity cost of overtaking is high since by focusing on a more exclusive matching market, matchmaker 2 could charge a much higher participation fee.

PROPOSITION 3.2. *If the type distribution is sufficiently diffused, there exists a pure-strategy equilibrium with a dual matching market structure in a sequential-move game.*

PROOF. Fix any $p_1 < a^2$. First, note that undercutting is dominated by overtaking for matchmaker 2. This is because in both cases, matchmaker 2 will serve all types, and overtaking generates a greater revenue with a higher price. It remains to show that for p_1 sufficiently small, it is not optimal for matchmaker 2 to drive matchmaker 1 out of the market by overtaking. By equation (2.1), the singular matching market structure S_2 obtains for any $p_2 \in (p_1, \theta(p_1)]$. The threshold type of participation is $c_2 = a$ because $p_1 < a^2$ implies $\theta(p_1) < a\mu(a,b)$. Matchmaker 2's revenue from overtaking is simply p_2 for any $p_2 \in (p_1, \theta(p_1)]$, so the best overtaking price is $\theta(p_1)$. For any $p_2 \in (\theta(p_1), \lambda(p_1))$, the dual matching market structure D_{12} obtains. By equation (2.3), $c_1 = a$, and c_2 satisfies $c_2(\mu(c_2,b) - \mu(a,c_2)) = p_2 - p_1$. Consider how matchmaker 2's revenue in the dual matching market structure D_{12}, given by $p_2(1 - F(c_2))$, changes at $p_2 = \theta(p_1)$. Since $c_2 = a$ at $p_2 = \theta(p_1)$, the derivative of matchmaker 2's revenue with respect to p_2 at $\theta(p_1)$ is positive if and only if

$$\mu(a,b) - a + a\left(f(a)(\mu(a,b) - a) - \frac{1}{2}\right) > f(a)\theta(p_1). \tag{3.1}$$

As p_1 approaches 0, $\theta(p_1)$ approaches $a(\mu(a,b) - a)$. Thus, the derivative is positive at $p_2 = \theta(p_1)$ for p_1 approaching 0, if and only if $\mu(a,b) > \frac{3}{2}a$. *Q.E.D.*

A sufficiently diffused distribution allows the first mover to survive the overtaking strategy of the second mover by focusing on a lower quality "niche" market. Note that the survival strategy of charging $p_1 < a^2$ for the first mover implies that all low types are served and some rents are left to the lowest type a with a relaxed participation constraint. Further, the sufficient condition of Proposition 3.2 depends on the type distribution only through the unconditional mean $\mu(a, b)$. This is because at the boundary between S_2 and D_{12} the behavior of matchmaker 2's revenue is independent of the type distribution, and is locally identical to that under the uniform type distribution.

3.2. Niche market

Proposition 3.2 provides a sufficient condition for the two matchmakers to coexist in an equilibrium, by considering the second mover's incentives to overtake the first mover when the latter charges a sufficiently low price. The analysis leaves open the possibility that, in equilibrium, both matchmakers have positive market shares and the first mover charges a higher price. We investigate this possibility in this subsection.

We will need a result about the revenue function of a one-price monopolist, $(1-F(c))p$, where c is determined by p through equation (2.1). In the appendix we prove that the revenue function is quasi-concave in price p (Lemma A.1). Let $\hat{p}$ be the solution to the one-price monopolist's revenue maximization problem. We have the following result.

Lemma 3.3. *Under the uniform type distribution, for any p_1 such that $\theta(p_1) > \hat{p}$, any best response p_2 of matchmaker 2 leaves zero revenue to matchmaker 1.*

Clearly the optimal response of matchmaker 2 is $\hat{p}$ if it is a feasible undercutting or overtaking price, which happens when $p_1 \in (\lambda(\hat{p}), b^2]$ and $p_1 \in (\theta^{-1}(\hat{p}), \hat{p})$ respectively. This leaves zero revenue to matchmaker 1. When $p_1 \in [\hat{p}, \lambda(\hat{p})]$, the maximum one-price monopolist revenue is not feasible. However, since $p_1 > \hat{p} > \theta^{-1}(\hat{p})$, overtaking matchmaker 1 dominates serving the higher quality market in a dual structure D_{12}. To see this, note that any price $p_2 \in (\theta(p_1), \lambda(p_1))$ that supports D_{12} leads to a duopolist's revenue, which is lower than the one-price monopolist's revenue at the same price p_2 because the latter has a greater market share. Since $\theta(p_1) > \hat{p}$, the quasi-concavity of the revenue function of the one-price monopolist implies that this is in turn lower than the overtaking revenue reached by charging $\theta(p_1)$. The proof of Lemma 3.3 uses the assumption of uniform type distribution to rule out serving the lower quality market in a dual structure D_{21} by showing that it is dominated by either overtaking or undercutting. In either case, matchmaker 1 gets zero revenue. The next result follows from Lemma 3.3 immediately.

Proposition 3.4. *Under the uniform type distribution, in any equilibrium with a dual matching market structure, the first mover serves the lower quality matching market.*

Proof. By Lemma 3.3, under the uniform type distribution, in any equilibrium with a dual matching market structure, we must have $p_1 \leq \theta^{-1}(\hat{p})$. We claim that matchmaker 2's best response p_2 belongs to the interval $[\theta(p_1), \lambda(p_1))$. The proposition then immediately follows this claim, because either there is no equilibrium with a dual market structure if $p_2 = \theta(p_1)$, or else D_{12} is the equilibrium market structure. To establish the claim, note that charging $p_2 \in [\lambda(p_1), b^2]$ cannot be optimal because matchmaker 2 would have zero revenue. If instead matchmaker 2 chooses $p_2 \in [0, \theta(p_1))$, there are most three possible scenarios. When the price pair (p_1, p_2) falls in the S_1 region, matchmaker 2 has no revenue. When the (p_1, p_2) falls in the S_2 region, matchmaker 2 is a monopolist. However at price $p_2 = \theta(p_1)$, matchmaker 2 is also a monopolist but has a higher revenue because $\theta(p_1) \leq \hat{p}$ and because the revenue function of the monopolist is quasi-concave. Finally, when (p_1, p_2) falls in the D_{21} region, matchmaker 2's revenue is lower than the revenue of a one-price monopolist at the same price p_2, which is lower than the revenue generated by charging $p_2 = \theta(p_1)$ due to the quasi-concavity. *Q.E.D.*

Proposition 3.4 shows that when matchmaker 1 chooses a low price p_1 such that $\theta(p_1) \leq \hat{p}$, matchmaker 2's best response is either charging the maximum overtaking price $\theta(p_1)$ or serving the higher quality market in a dual structure D_{12}. Note that this result holds regardless of the type distribution. While Lemma 3.3, and hence the conclusion of Proposition 3.4, does depend on the assumption of the uniform type distribution, the intuition behind it is more general. By charging a very low price, or a very high price, the first mover targets a niche market of few types and makes the overtaking strategy unappealing to the second mover. However, while a high price might invite undercutting, a low price is less vulnerable. Indeed, under the uniform type distribution, if the first mover charges a price high enough to deter overtaking, the second mover will find it optimal to undercut. Thus, to deter undercutting as well as overtaking, matchmaker 1 has to find its niche market with low prices.

3.3. Market coverage

To study the effects on competition, we compare our duopoly model with a two-price monopoly matchmaker. This is a natural comparison because the number of potential matching markers is two in both cases. The monopolist's problem can be stated as choosing two participation thresholds, c_1 and c_2, with $c_1 \leq c_2$, to maximize the total revenue:[11]

$$(1 - F(c_1))c_1\mu(c_1, c_2) + (1 - F(c_2))c_2(\mu(c_2, b) - \mu(c_1, c_2)). \tag{3.2}$$

In this subsection we consider how competition affects the total matching market coverage, i.e. the lower participation threshold.

PROPOSITION 3.5. *In any equilibrium with the dual structure D_{12} the market coverage is at least as large as in the optimal structure of a monopolist if matchmaker 2's revenue is quasi-concave in c_2 for any p_1.*

PROOF. Rewrite the revenue of the monopolist (equation 3.2) as

$$((1 - F(c_1))c_1 - (1 - F(c_2))c_2)\mu(c_1, c_2) + (1 - F(c_2))c_2\mu(c_2, b). \tag{3.3}$$

Since the first term in the above expression can be made arbitrarily small with c_1 just below c_2, the optimal thresholds $\hat{c}_1$ and $\hat{c}_2$ satisfy

$$(1 - F(\hat{c}_1))\hat{c}_1 \geq (1 - F(\hat{c}_2))\hat{c}_2. \tag{3.4}$$

Differentiating (3.3) with respect to c_1, and assuming an interior $\hat{c}_1$, we find:

$$\frac{1 - F(\hat{c}_2)}{F(\hat{c}_2) - F(\hat{c}_1)}(\hat{c}_2 - \hat{c}_1) = \frac{\rho(\hat{c}_1)\mu(\hat{c}_1, \hat{c}_2) - \hat{c}_1^2}{\mu(\hat{c}_1, \hat{c}_2) - \hat{c}_1}.$$

[11] The proof of Lemma 3.10 in the appendix shows that the monopolist will always choose prices such that a dual matching market structure obtains. Thus, applying the selection criterion introduced in section 2 we can use participation thresholds (as opposed to prices) as choice variables for the monopolist's revenue-maximization problem.

If $\rho(\hat{c}_1) > \hat{c}_1$, then the right-hand-side of the above condition is greater than $\hat{c}_1$, resulting in an inequality that contradicts (3.4). Thus, $\rho(\hat{c}_1) \leq \hat{c}_1$.

For duopolistic coverage, matchmaker 2 chooses c_2 to maximize its revenue

$$(1 - F(c_2))(p_1 + c_2(\mu(c_2, b) - \mu(c_1, c_2))) \tag{3.5}$$

subject to

$$\begin{cases} c_1\mu(c_1, c_2) = p_1 & \text{if} \quad c_1 > a; \\ a\mu(a, c_2) \geq p_1 & \text{if} \quad c_1 = a. \end{cases} \tag{3.6}$$

It suffices to consider the case where c_1 is greater than a and determined by (3.6) at some equilibrium price $p_1 = \tilde{p}_1$. Taking derivatives of (3.6) we have

$$\frac{\mathrm{d}c_1}{\mathrm{d}c_2} = -\frac{c_1\mu_r(c_1, c_2)}{\mu(c_1, c_2) + c_1\mu_l(c_1, c_2)}. \tag{3.7}$$

Since matchmaker 2's revenue function is quasi-concave in c_2, a necessary condition for equilibrium is that matchmaker 2's revenue increases with c_2 at the boundary between S_2 and D_{12} where $c_2 = c_1$ and $p_2 = \theta(\tilde{p}_1)$. At this point, equation (3.7) becomes $\mathrm{d}c_1/\mathrm{d}c_2 = -\frac{1}{3}$ as $\mu_l = \mu_r = \frac{1}{2}$ at the boundary, and by taking derivatives of (3.5) we find that this necessary condition is satisfied if and only if at some equilibrium lower threshold $\tilde{c}_1 > a$,

$$\rho(\tilde{c}_1)\left(\mu(\tilde{c}_1, b) - \frac{4}{3}\tilde{c}_1\right) > \tilde{c}_1^2. \tag{3.8}$$

Since $\mu_l(c, b) \leq 1$ by Assumption 2.2, (2.6) implies that for any c we have

$$\mu(c, b) - c \leq \rho(c). \tag{3.9}$$

Thus, condition (3.8) can be satisfied only if $\rho(\tilde{c}_1) > \tilde{c}_1$. The proposition then follows immediately from Assumption 2.2. *Q.E.D.*

Without the assumption of quasi-concavity, condition (3.8) is generally not necessary for an equilibrium dual market structure, because matchmaker 2's revenue may decrease with c_2 at the boundary between S_2 and D_{12} and yet there is a price p_2 in the D_{12} region that dominates any overtaking price. In the appendix we show that, under uniform type distribution, matchmaker 2's revenue is globally concave in c_2 for any p_1 in the D_{12} region

(Lemma A.2). Moreover, by Proposition 3.4, under the same assumption any equilibrium dual market structure is D_{12} . Thus we have the following result.

Corollary 3.6. *If the type distribution is uniform, then in any equilibrium with a dual matching market structure the market coverage is at least as large as in the optimal structure of a monopolist.*

The intuition behind the above result is more general than implied by the uniform type distribution. Competition expands the total market coverage because of the need for the first mover to survive price competition. Only by lowering its price sufficiently and catering to a low quality matching market can the first mover prevent overtaking.

3.4. Market differentiation

In this subsection, we ask how the equilibrium market differentiation, in terms of how exclusive the high quality matching market is, compares with the optimal market differentiation that maximizes the total revenue for the two-price monopolist.

Definition 1. *A dual matching market structure with $c_1 < c_2$ has a greater conditional market differentiation than another one with $c'_1 < c'_2$ if $c_1 = c'_1$ and $c_2 > c'_2$.*

Definition 1 limits our comparison of matching market structures to those with the same market coverage. We drop the qualifier "conditional" when there is no risk of confusion. Market differentiation here does not refer to the comparison in terms of the quality difference $m_2 - m_1$ between the two markets. Instead, it describes how exclusive the high quality matching market is: a dual matching market structure has a greater differentiation if $c_2(m_2 - m_1)$ is greater, or in words if the higher threshold c_2 is willing to pay more for the quality difference between the two matching market. Under Assumption 2.1, the quality difference $m_2 - m_1$ is non-decreasing in c_2 for fixed c_1, so this interpretation of exclusivity coincides with Definition 1 above. Note that market differentiation in terms of exclusivity, is what matters to revenue maximization for the two-price monopolist and the duopolist that serves the high quality matching market. We have the following comparison result:

PROPOSITION 3.7. *In any equilibrium with the dual structure D_{12} the equilibrium outcome has less market differentiation than the optimal structure of a monopolist.*

PROOF. The monopolist's differentiation problem is to choose c_2 to maximize (3.2) taking as given c_1, subject to the constraint (3.6). The first order condition can be written as

$$\frac{\mu(\hat{c}_2, b) - \mu(c_1, \hat{c}_2)}{\hat{c}_2 - \mu(c_1, \hat{c}_2)} = \frac{1 - F(c_1)}{\rho(\hat{c}_2)} \frac{\hat{c}_2 - c_1}{F(\hat{c}_2) - F(c_1)}. \tag{3.10}$$

The right-hand-side of (3.10) approaches 1 while the left-hand-side becomes arbitrarily large when $\hat{c}_2$ takes on the value of c_1, and the opposite happens when $\hat{c}_2$ approaches b. Thus, for any c_1, there exists at least one $\hat{c}_2$ that satisfies (3.10). Further, the right-hand-side is increasing $\hat{c}_2$, because Assumption 2.1 implies $(\hat{c}_2 - c_1)/(F(\hat{c}_2) - F(c_1))$ increases with $\hat{c}_2$ while $\rho(\hat{c}_2)$ decreases with $\hat{c}_2$ by Assumption 2.2. The left-hand-side decreases in $\hat{c}_2$, because $\mu_l(\hat{c}_2, b) \leq 1$ by Assumption 2.2. Thus, a unique $\hat{c}_2$ satisfies (3.10).

For duopolistic differentiation, matchmaker 2 chooses c_2 to maximize (3.5) taking as given p_1, subject to (3.6). The first order necessary condition can be written as

$$\frac{\rho(\tilde{c}_2)(\mu(\tilde{c}_2, b) - \mu(c_1, \tilde{c}_2))}{\tilde{c}_2 - \mu(c_1, \tilde{c}_2)} = \frac{1 - F(c_1)}{F(\tilde{c}_2) - F(c_1)} \left(1 + \frac{\mu(c_1, \tilde{c}_2) - c_1}{\tilde{c}_2 - \mu(c_1, \tilde{c}_2)} \frac{\mathrm{d}c_1}{\mathrm{d}c_2}\right) \tilde{c}_2 + \frac{c_1 \mu(c_1, \tilde{c}_2)}{\tilde{c}_2 - \mu(c_1, \tilde{c}_2)},$$

where $\mathrm{d}c_1/\mathrm{d}c_2$ is given by (3.7). By Assumption 2.1, we have $\mu(c_1, \tilde{c}_2) - c_1 \leq \tilde{c}_2 - \mu(c_1, \tilde{c}_2)$. Moreover, $\mu_r(c_1, c_2) \leq \frac{1}{2} \leq \mu_l(c_1, c_2)$. Thus, the first order condition implies

$$\frac{\rho(\tilde{c}_2)(\mu(\tilde{c}_2, b) - \mu(c_1, \tilde{c}_2))}{\tilde{c}_2 - \mu(c_1, \tilde{c}_2)} > \frac{1 - F(c_1)}{F(\tilde{c}_2) - F(c_1)} \tilde{c}_2 - \frac{1 - F(c_2)}{F(\tilde{c}_2) - F(c_1)} \frac{\tilde{c}_2 c_1}{2\mu(c_1, \tilde{c}_2) + c_1}. \tag{3.11}$$

Comparing (3.11) and (3.10), we find that $\tilde{c}_2 < \hat{c}_2$ for any c_1 if

$$(2\mu(c_1, \tilde{c}_2) + c_1)(1 - F(c_1)) - (1 - F(\tilde{c}_2))\tilde{c}_2 \geq 0.$$

Note that the above inequality holds at $\tilde{c}_2 = c_1$. Further, the derivative of the left-hand-side with respect to $\tilde{c}_2$ has the same sign as

$$2(1 - F(c_1)) \frac{\tilde{c}_2 - \mu(c_1, \tilde{c}_2)}{F(\tilde{c}_2) - F(c_1)} + \tilde{c}_2 - \rho(\tilde{c}_2).$$

The above is strictly positive as $\tilde{c}_2$ approaches c_1 from above, and is strictly increasing in $\tilde{c}_2$ because $(\tilde{c}_2 - \mu(c_1, \tilde{c}_2)/(F(\tilde{c}_2) - F(c_1))$ is weakly increasing in $\tilde{c}_2$ by Assumption 2.1, while $\rho(\tilde{c}_2)$ is weakly decreasing by Assumption 2.2. *Q.E.D.*

Definition 1 requires us to compare monopolistic and duopolistic differentiation for fixed market coverage. The above proof establishes that for any equilibrium coverage, at the optimal choice of differentiation $\tilde{c}_2$ of matchmaker 2, the monopolist's revenue is strictly increasing in c_2. Since it uses only the first order necessary condition, the proof does not require the assumption that matchmaker 2's revenue is quasi-concave. Under uniform type distribution we can strengthen the proposition.

Corollary 3.8. *If the type distribution is uniform, in any equilibrium with the dual matching market structure the equilibrium outcome has less market differentiation than the optimal structure of a monopolist.*

Intuitively, when choosing its own price, matchmaker 2 does not internalize the cannibalization of the lower market. Rewrite the monopolist's revenue function (3.2) as

$$(F(c_2) - F(c_1))p_1 + (1 - F(c_2))(p_1 + c_2(\mu(c_2, b) - \mu(c_1, c_2))), \tag{3.12}$$

and compare it with matchmaker 2's objective function (3.5). Since c_1 either stays constant at a or increases as c_2 decreases according to (3.6), the first term that appears in (3.12) but is absent from (3.5) means that duopolist matchmaker 2 has a greater incentive to lower c_2 relative to the monopolist.[12] Such incentive exists regardless of the type distribution. The uniform distribution assumption is used to ensure that the equilibrium matching market structure is D_{12} by way of Proposition 3.4.

3.5. Welfare comparison

To complete the comparison between duopolistic matchmaking and monopolistic matchmaking, we now examine the welfare in terms of the total match value, given by

$$(F(c_2) - F(c_1))\mu^2(c_1, c_2) + (1 - F(c_2))\mu^2(c_2, b), \tag{3.13}$$

for any pair of participation thresholds c_1 and c_2 with $c_1 \leq c_2$. A useful benchmark for the comparison is the two-market planner's problem, which is to choose the efficient thresholds

[12] The proof of Proposition 3.7 is complicated by the fact that the constraint (3.6) has different implications to the monopolist and the duopolist matchmaker 2: the former chooses c_2 for fixed c_1 with p_1 determined by (3.6), while the latter chooses c_2 for fixed p_1 with c_1 determined by the same constraint.

c_1^* and c_2^*, with $c_1^* \leq c_2^*$, to maximize the total match value (3.13).[13] First, we compare optimal coverage $\hat{c}_1$ and efficient coverage c_1^*.

LEMMA 3.9. *Monopolistic market coverage is at most the efficient coverage.*

The above result that the monopolist's matching markets are smaller and more selective than the planner's does not require the assumption of uniform type distribution. In particular, following the standard price discrimination literature, we can define "virtual type" of x as $x - \rho(x)$. As shown in Proposition 3.5, the monopolist will never serve agents of negative virtual types, establishing that the optimal coverage $\hat{c}_1$ satisfies $\hat{c}_1 \geq \rho(\hat{c}_1)$. In contrast, the planner will service additional low types so long as the benefit from the expansion of the market coverage is not outweighed by the loss due to the reduction in the average quality of the lower matching market. The proof of Lemma 3.9 shows instead $c_1^* < \rho(c_1^*)$ whenever $c_1^* > a$, implying that $\hat{c}_1 \leq c_1^*$ by Assumption 2.2. Next, we compare the optimal market differentiation $\hat{c}_2$ for the two-price monopolist with the efficient differentiation c_1^* for the two-market planner under any total coverage c_1.

LEMMA 3.10. *Monopolistic differentiation is efficient if the type distribution is uniform.*

For both the planner and the monopolist, increasing c_2 raises the quality in both matching markets at the expense of reducing the relative size of the higher quality market. The effect on the objective functions is generally different because the monopolist is concerned with the change in the marginal type's willingness to pay, whereas the planner cares about the change in the average expected type. Lemma 3.10 shows that the effect is the same for type distributions with a linear conditional mean function $\mu(\cdot, b)$, including uniform and exponential distributions.

Since by Lemma 3.10 the monopolist and the planner have identical incentives for market differentiation under the uniform type distribution, Corollary 3.8 implies that competition between the two matchmakers induces a smaller, and less efficient, degree of market differentiation. On the other hand, Lemma 3.9 establishes that the monopolist has an

[13] We implicitly assume that the planner is restricted to threshold participation strategies. This may be motivated by the assumption that the planner faces the same informational constraints.

inefficiently small market coverage, and therefore by Corollary 3.6 duopolistic matchmaking may correct this distortion. The trade-off between differentiation and coverage then implies that the welfare comparison between duopolistic matchmaking and monopolistic matchmaking in terms of the total match value can go either way. The comparison generally depends on how diffused the type distribution is. For the uniform type distribution, the degree of diffusion is determined by the value of a/b, with a lower value of the ratio corresponding to a more diffused distribution. We have the following result.

PROPOSITION 3.11. *If the type distribution is uniform, duopolistic matchmaking generates a smaller total match value than monopolistic matchmaking if and only if the diffusion of the type distribution falls below a critical value.*

PROOF. Under the uniform type distribution, the total match value (3.13) is given by

$$R(c_1, c_2) = \frac{1}{4(b-a)} \left((c_2 - c_1)(c_1 + c_2)^2 + (b - c_2)(c_2 + b)^2 \right).$$

Note that the comparison between the total match value $\tilde{R}$ under duopolistic matchmaking and $\hat{R}$ under monopolistic matchmaking depends on a only through its effects on the equilibrium thresholds $\tilde{c}_1$ and $\tilde{c}_2$ versus the monopolist's optimal thresholds $\hat{c}_1$ and $\hat{c}_2$.

Under the two-price monopolist, the optimal thresholds $\hat{c}_1$ and $\hat{c}_2$ can be solved from the first order conditions with respect to c_1 and c_2, derived from (3.2). This yields $\hat{c}_1 = \max\{a, h\}$ and $\hat{c}_2 = \frac{1}{2}(\hat{c}_1 + b)$, with $h = b(2\sqrt{6}+3)/15$. Since $h > \frac{1}{2}b$, we have that $\hat{c}_1$ and $\hat{c}_2$ are constant in a for any $a/b < \frac{1}{2}$. The total match value $\hat{R}$ is then $R(\hat{c}_1, \hat{c}_2)$.

For duopolistic matchmaking, we distinguish three cases. In the first case, a/b lies between $\sqrt{19} - 4$ and $\frac{1}{2}$. By Propositions 3.2 and 3.4, an equilibrium with a dual market structure D_{12} exists. Since a necessary condition for an equilibrium with a lower threshold $\tilde{c}_1 > a$ is condition (3.8), which under the uniform distribution becomes $\tilde{c}_1 < (\sqrt{19} - 4)b$, the equilibrium satisfies $\tilde{c}_1 = a$. In this case, the equilibrium higher threshold $\tilde{c}_2$ can be computed explicitly by backward induction: the best response of matchmaker 2 to any p_1 is $c_2 = b/2 - p_1/(b-a)$, the equilibrium price for matchmaker 1 is $\tilde{p}_1 = \frac{1}{4}(b-2a)(b-a)$, and finally $\tilde{c}_2 = \frac{1}{4}(b+2a)$. The total match value $\tilde{R}$ for a/b between $\sqrt{19}-4$ and $\frac{1}{2}$ is then $R(a, \frac{1}{4}(b+2a))$. Comparing $\tilde{R}$ with $\hat{R}$, we find that there is a critical value of a/b between $\sqrt{19} - 4$ and $\frac{1}{2}$, such that $\tilde{R} < \hat{R}$ if and only if a/b is greater than the critical value.

In the second case, a/b is smaller than $\sqrt{19} - 4$. Explicit formulae for $\tilde{c}_1$ and $\tilde{c}_2$ are not easily obtained because we may have $\tilde{c}_1 > a$, but we make the following two observations. First, if $\tilde{c}_1 > a$, then $\tilde{c}_1$ and $\tilde{c}_2$ are both independent of a. This is because under the uniform type distribution, equations (3.5) and (3.6) imply that matchmaker 2's best response does not depend on a, which in turn implies that matchmaker 1's problem of $\max_{p_1}(F(c_2) - F(c_1))c_1\mu(c_1, c_2)$ does not depend on a either. Second, the equilibrium thresholds $\tilde{c}_1$ and $\tilde{c}_2$ are continuous in the parameter a/b. Hence for some $a/b \leq \sqrt{19}-4$, we have $\tilde{R} = R(\tilde{c}_1, \tilde{c}_2) = R(a, \frac{1}{4}(b+2a))$, which is strictly greater than $\hat{R}$ by direct calculation. Since $\tilde{c}_1$ and $\tilde{c}_2$ do not depend on a, we have $\tilde{R} > \hat{R}$ for all a/b such that $\tilde{c}_1 > a$.

In the third case, we have $a/b \geq \frac{1}{2}$. In the appendix, we prove that there is no equilibrium with a dual market structure (Lemma A.3). Then, there is a continuum of equilibria indexed by the price charged by the first mover. In any such equilibrium, the total match value does not exceed the value achieved by a one-market planner that solves $\max_c(1 - F(c))\mu^2(c, b)$. Under the uniform type distribution, the planner's solution is $c^* = a$. Thus, the maximum total match value $\tilde{R}$ under duopolistic matchmaking is $R(a, a)$. It is easily verified that $\hat{R} > R(a, a)$ for all $a/b \geq \frac{1}{2}$. *Q.E.D.*

The reason that the equilibrium outcome is less efficient in sorting than the monopoly outcome when the type distribution is not too diffused is best understood for the intermediate values of a/b. In this range (more precisely, when a/b lies between $\sqrt{19} - 4$ and $\frac{1}{2}$), it is efficient to serve all types (i.e. $c_1^* = a$). The monopolistic coverage is inefficiently small with $\hat{c}_1 > a$, while its differentiation is efficient. In contrast, the equilibrium outcome has the efficient market coverage with $\tilde{c}_1 = a$, but suffers from inefficiently small market differentiation. When a/b is large in this range, the loss from insufficient coverage under monopoly is small relative to the gain in efficient differentiation, because the optimal coverage becomes close to the efficient coverage. As a result, sorting is more efficient overall under monopoly than under competition. The intuition is similar for extreme values of diffusion of the type distribution. Indeed, the trade-off between coverage and differentiation disappears when a/b is sufficiently high (more precisely, if a/b is greater than h, defined in the above proof), as the monopolistic coverage is efficient while the duopolistic differentiation is none because the first mover cannot survive overtaking.

3.6. Discussions

Our results comparing duopolistic sorting and monopolistic sorting in terms of market differentiation and market coverage are obtained under the assumption of uniform type distribution, although both the non existence of pure-strategy equilibrium in the simultaneous pricing game (Lemma 3.1) and the sufficient condition for existence of dual market structure in the sequential pricing game (Proposition 3.2) hold more generally. Among non uniform distributions of particular interest is the exponential distribution, which has a density function $\exp(-(x-a)/\beta)/\beta$ with $a, \beta > 0$, and which satisfies Assumptions 2.1 and 2.2. Since the support of the type distribution is unbounded, the upper bound function λ is undefined and undercutting is not feasible. As a result, matchmaker 1 can always survive overtaking by charging a price p_1 sufficiently high so that it becomes more profitable for matchmaker 2 to serve low types in the dual market structure D_{21}. Indeed, we can show that when $a/\beta > 2$, the equilibrium matching market structure is D_{21}.[14] Intuitively, serving low types is lucrative when a is great and the type distribution is tightly concentrated on these types (i.e. β is small). In this case, the first mover would be overtaken by the second mover if it tries to compete for low types. This forces the first mover to serve a niche market of high types. In this kind of equilibrium, we expect differentiation to be greater under competition than under monopoly, as the second mover does not internalize the negative impact on the size of the first mover's market in increasing the lower participation threshold. As the exponential distribution has a linear conditional mean function, our result about the efficiency of monopolistic market differentiation continues to hold (Lemma 3.10). Thus, duopolistic differentiation remains less efficient compared to monopoly matchmaking.

A restriction in the present model of competing matchmaking is that each matchmaker is allowed to use only one price and create one matching market. We have made

[14] This inequality is exactly the opposite of the condition in Proposition 3.2 for the exponential case. When $a/\beta > 2$, for any price p_1 below a^2, the maximum revenue for matchmaker 2 as a duopolist in D_{12} is reached at the boundary between D_{12} and S_2, leaving zero revenue for matchmaker 1. Moreover, in this parameter range, the one-price monopolist's optimal price is $\hat{p} = a(a+\beta)$, and so matchmaker 2's best response to $p_1 \in (a^2, \hat{p})$ is to overtake with price $p_2 = \hat{p}$, leaving zero revenue for matchmaker 1. Finally, for prices p_1 above $\hat{p}$, the best response of matchmaker 2 is either to be a duopolist in D_{21} with a price $p_2 < \theta^{-1}(p_1)$, or to overtake with a price p_2 just above p_1.

the assumption to simplify the analysis.[15] The results comparing the monopolist and the planner in terms of matching market coverage and differentiation turn out to be robust to the restriction to two matching markets. In an earlier paper (Damiano and Li, forthcoming), we consider the problem of a monopoly matchmaker that uses a schedule of entrance fees to sort different types of agents on the two sides of a matching market into exclusive matching markets, where agents randomly form pairwise matches. That paper has a more general setup than the model of monopolistic sorting in the present paper, with asymmetric type distributions and an unrestricted number of matching markets.[16] By the results of Damiano and Li (forthcoming), in the present paper Assumption 2.2 is sufficient to imply that the monopolist unconstrained in the number of matching markets has the same incentive as the planner to perfectly sort all participating types (i.e. one market for each participating type), while the market coverage for the monopolist is at most as large as the efficient full coverage for the planner. Further, it is straightforward to establish that under the uniform type distribution for any finite number of matching markets that can be offered, total market coverage is at least as large for the planner as for the monopolist, and monopolistic market differentiation is efficient given the total market coverage. For price competition, it turns out that the result of inefficient sorting under competition (Corollary 3.8) is robust, but the extent of sorting inefficiency depends on the number of matching markets. In the extreme case when matchmakers can create an arbitrarily large number of matching markets, and hence perfect sorting of all agents is possible, price competition would not lead to inefficient sorting, because the type distribution in each matching market is degenerate and the overtaking strategy completely loses its power. However, as long as types are not perfectly sorted, overtaking is possible and price competition interferes with sorting. When choosing their pricing structure, each matchmaker fails to internalize its effect on the market share of the competitors, thus leading to sorting inefficiency.

[15] McAfee (2002) shows most of the efficiency gains in sorting can be made with a total of just two matching markets. He does not consider the incentives of market participants.

[16] Rayo (2002) studies how a monopolist can use price discrimination to sell status goods. His problem can be interpreted as a special case of the matching model of Damiano and Li (forthcoming) by assuming that the two sides have identical type distributions.

4. Concluding Remarks

Sorting of heterogeneous types is an essential ingredient in the literature on unintermediated matching markets in that match formation decisions of participants in a matching market depend on the distribution of types in the market (Burdett and Coles, 1997; Shimer and Smith, 2000; Damiano, Li and Suen, 2005). However, as already discussed in the introduction, the existing literature on competing intermediaries in matching markets has so far ignored the issue of sorting, with the exclusive focus on the size effects. More broadly, some recent papers on directed search and competitive search markets allow for type heterogeneity, but since there is no complementarity in a buyer-seller or worker-firm matching market, prices do not play any sorting role (see Montgomery, 1991; Mortensen and Wright, 2002; Inderst, 2005). By introducing type heterogeneity into a model of competing matchmakers, we highlight a role of prices in coordinating participants' market decisions and determining match qualities, and derive important implications of price competition to sorting efficiency. Our results on the potential inefficiencies of price competition do not suggest that competition is necessarily harmful or that monopoly is always desirable, but they do mean that regulatory policies in a matching environment should not be exclusively focused on enhancing price competition so as to expand market coverage. Attention must also be paid to how price competition interacts with the sorting of heterogeneous agents. Gains from expanding market coverage to additional low types may need to be weighed against loss resulting from less efficient sorting.

When perfect sorting is not feasible, price competition interferes with the sorting role of prices. How compelling is the assumption of imperfect sorting ultimately depends on how heterogeneous we think agents are. If only a few types of agents can be profitably distinguished, perfect sorting is likely to be feasible and the benefits of competition in terms of greater market coverage will tend to outweigh any sorting inefficiency. In contrast, when the type space is very rich, it is unlikely that sufficiently many matching markets can be created to perfectly sort all agents, either because the cost of market creation is too high or because the presence of some size effect makes thin markets unattractive. In this environment the benefits from sorting are large, and we may expect a monopolist to induce a more efficient matching market structure than competing matchmakers.

Appendix

A.1. Kohlberg and Mertens Stability and Proof of Lemma 2.3

In this subsection we define the notion of a stable collection of matching market structures in the sense of Kohlberg and Mertens (1986) and prove Lemma 2.3. Fix any p_1 and p_2 with $p_1 < p_2$, and consider the simultaneous-move game of agents choosing whether to participate in matching market 1 or 2, or not to participate. Let γ^ϵ be a perturbed game where for each type x and each of the three participation choices some fraction strictly between 0 and ϵ of type x agents is constrained to that choice.

DEFINITION 2. *A collection T of matching market structures is stable if: i) for any $\eta > 0$, there exists an $\hat{\epsilon} > 0$ such that for all $\epsilon < \hat{\epsilon}$, any game γ^ϵ has a Nash equilibrium in which at most a fraction η of all agents makes a participation choice different from the one made in some matching market structure in T; ii) no strict subset of T satisfies property i).*

To prove Lemma 2.3, we consider only the case of $p_2 > \lambda(p_1)$ where we show that the unique stable collection is a singleton that contains S_1; the other two cases can be similarly proved. First, we show that as ϵ becomes small, each perturbed game γ^ϵ has a Nash equilibrium that is arbitrarily close to S_1 in terms of participation decisions. Let m_1^ϵ and m_2^ϵ the mean quality of the agents constrained to participating in matching market 1 and 2 respectively in γ^ϵ. For each $i = 1, 2$, denote as $\mu_i^\epsilon(t, t')$ the conditional mean in the interval (t, t') after excluding the agents constrained to not participating or to participating in market $j \neq i$. Let c_1^ϵ solve (2.1), with μ_1^ϵ in place of the conditional mean in market 1. Consider the strategy profile in which unconstrained agents of types lower than c_1^ϵ do not participate while all other unconstrained agents participate in matching market 1. Take any sequence of games $\{\gamma^\epsilon\}_{\epsilon \to 0}$. For any such sequence and any pair of threshold $t < t'$, we have $\mu_1^\epsilon(t, t')$ converges to $\mu(t, t')$. For ϵ small, c_1^ϵ is close to the solution to (2.1). Since $m_2^\epsilon \leq b$ and $p_2 > \lambda(p_1)$, from the definition of $\lambda(p_1)$ we have $b(m_2^\epsilon - \mu_1^\epsilon(c_1^\epsilon, b)) + p_1 < p_2$ for ϵ sufficiently small. Then, the proposed strategy profile is a Nash equilibrium of γ^ϵ, and it converges to S_1 as ϵ converges to 0.

Next, we show that for any ϵ sufficiently small, there exists some game γ^ϵ that does not have a Nash equilibrium close to any other matching market structure. Consider a sequence of games $\{\gamma^\epsilon\}_{\epsilon\to 0}$ where both m_1^ϵ and m_2^ϵ are close to b for all ϵ. (This is possible because we can make the probability of tremble for the highest type converge to 0 infinitely slower than all other types.) To rule out D_{12}, suppose that for each γ^ϵ, there is a Nash equilibrium in which both matchmakers have a strictly positive market share, and let $c_1^\epsilon < c_2^\epsilon < b$ be the thresholds in such equilibrium. Then c_1^ϵ and c_2^ϵ must solve equations (2.2) or (2.3), with μ_1^ϵ and μ_2^ϵ in place of the conditional means in the two markets. However, since $p_2 > \lambda(p_1)$, as ϵ becomes small, neither of the above systems of equations has a solution, a contradiction. To rule out S_2, suppose that there is a sequence of equilibria for ϵ going to zero, such that in the limit only matchmaker 2 has a positive market share. In such sequence, the marginal participating type c_2^ϵ in market 2 must converge to the solution to $c_2\mu(c_2, b) = p_2$. As ϵ becomes small, the quality of matching market 1 can be either m_1^ϵ or $\mu_1^\epsilon(c_1^\epsilon, c_2^\epsilon)$ with c_1^ϵ converging to c_2^ϵ, or somewhere in between. Since $p_2 > \theta(p_1)$, for ϵ sufficiently small c_2^ϵ will strictly prefer joining market 1, a contradiction. Finally, to rule out the null market structure, suppose that there is a sequence of equilibria such that in the limit neither matchmaker has a positive market share. In any such sequence, market 1's quality is m_1^ϵ. Since m_1^ϵ is arbitrarily close to b and $p_1 < b^2$, for ϵ sufficiently small, the highest type agents will strictly prefer joining market 1 to not participating, a contradiction.

A.2. Proof of Lemma 3.1

First, note that only a dual matching market structure with strictly positive revenues for both matchmakers is a candidate for equilibrium outcome. This is because for any competitor's price, say p_1, the other matchmaker can earn a strictly positive revenue by either overtaking or undercutting.

Second, in any dual matching market structure, by using the overtaking strategy each matchmaker can earn a revenue strictly greater than the competitor, which is impossible. To see this point, without loss of generality suppose that $0 < p_1 \leq p_2 < b^2$ and consider the dual matching market structure D_{12}. The participation threshold c_1 and c_2 are determined by equation (2.2), or in the case of $c_1 = a$, by equation (2.3). If matchmaker 2 charges a price just above p_1, say $p_1 + \epsilon'$, then in the case of $c_1 > a$, matchmaker 2 becomes a monopolist and the participation threshold c' is determined as in (2.1). Comparing the two equations $c'\mu(c', b) = p_1 + \epsilon'$ and $c_1\mu(c_1, c_2) = p_1$, we conclude that $c' < c_1$ for some ϵ' slightly greater than zero. In the case of $c_1 = a$, matchmaker 2 becomes a monopolist by charging a price just above p_1, and the participation threshold $c' = a$. In either case, matchmaker 2 earns a strictly greater revenue in deviation than matchmaker 1 does in the dual matching market structure through a higher price and a larger matching market.

Similarly, given p_2, if matchmaker 1 overtakes with a price just above p_2, say $p_2 + \epsilon''$, then matchmaker 1 becomes a monopolist and the participation threshold c'' is determined as in (2.1). Since $c_2\mu(c_2, b) = p_2 + c_2\mu(c_1, c_2) - p_1 > p_2 + c_1\mu(c_1, c_2) - p_1 \geq p_2$, for some ϵ'' slightly greater than zero, we have $c'' < c_2$. Thus, matchmaker 1 earns a strictly greater revenue in deviation than matchmaker 2 in the dual market structure.

A.3. Lemma A.1 and Proof

LEMMA A.1. *The revenue function of a one-price monopolist is quasi-concave in price p.*

PROOF. Consider the equivalent problem of choosing a threshold $c \geq a$ to maximize $(1 - F(c))c\mu(c, b)$. If the optimal threshold $\hat{c}$ is interior then it satisfies the first order condition $\rho(\hat{c})\mu(\hat{c}, b) - \hat{c}^2 = 0$. Since $\mu_l(c, b) \leq 1$ and $\rho'(c) \leq 0$, the derivative of $\rho(c)\mu(c, b) - c^2$ is less than $\rho(c) - 2c$, which is less than 0 at any $\hat{c}$ that satisfies the first order condition. It follows that the monopolist's revenue function is quasi-concave in c. Since the revenue is simply p for $p < a\mu(a, b)$ and there is a one-to-one relation between c and p for $p \geq a\mu(a, b)$ (given by 2.1), the revenue function is also quasi-concave in p.

A.4. Proof of Lemma 3.3

We only need to show that for any $p_1 \in [\hat{p}, \lambda(\hat{p})]$, any best response of matchmaker 2 leaves zero revenue to matchmaker 1. For any such price p_1, matchmaker 2 has at most four viable options. (i) Matchmaker 2 can overtake by charging $p_2 \in (p_1, \theta(p_1)]$. By quasi-concavity, matchmaker 2's maximum overtaking revenue is $(1 - F(c_2))p_1$, with a price p_2 arbitrarily close to p_1, and c_2 satisfying $c_2\mu(c_2, b) = p_1$ by (2.1). (ii) Matchmaker 2 can undercut by charging $p_2 \in [0, \lambda^{-1}(p_1)]$ (when $\lambda^{-1}(p_1)$ is defined). Since $p_1 \leq \lambda(\hat{p})$, by the quasi-concavity the maximum undercutting revenue is $(1 - F(c_2))\lambda^{-1}(p_1)$ obtained by charging $p_2 = \lambda^{-1}(p_1)$, where c_2 satisfies $c_2\mu(c_2, b) = \lambda^{-1}(p_1)$ by (2.1). (iii) Matchmaker 2 can allow the dual structure D_{12} by charging $p_2 \in [\theta(p_1), \lambda(p_1)]$. However, this option is dominated by the option of overtaking by the quasi-concavity. (iv) Matchmaker 2 can allow the dual structure D_{21} by charging $p_2 \in (\lambda^{-1}(p_1), \theta^{-1}(p_1))$. We want to use the assumption of uniform type distribution to show that option (iv) is never optimal because it is dominated by either overtaking or undercutting. Note that the maximum overtaking revenue decreases with p_1, while the maximum undercutting revenue increases in p_1. In addition, because for fixed $p_2 < p_1$ as p_1 increases c_2 either decreases or does not change and c_1 increases (see equations (2.2) and (2.3), with the roles of the two matchmakers reversed), matchmaker 2's maximum revenue in D_{21} is increasing in p_1.

The argument for ruling out $p_2 \in (\lambda^{-1}(p_1), \theta^{-1}(p_1))$ relies on two claims. The first is that there is a critical price $\overline{p}$ such that for any $p_1 \geq \overline{p}$ matchmaker 2's maximum revenue as a duopolist is achieved at the boundary between S_2 and D_{21}. Then, the maximum revenue as a duopolist coincides with the maximum undercutting revenue for any $p_1 \geq \overline{p}$, with zero revenue for matchmaker 1. The second claim is that at $p_1 = \overline{p}$ the maximum undercutting revenue is smaller than the maximum overtaking revenue. For any $p_1 < \overline{p}$, the maximum revenue as a duopolist is achieved in the interior of the D_{21} region. However, for fixed p_2, the revenue to matchmaker 2 in D_{21} is increasing in p_1, and so its maximum revenue is also increasing in p_1. Since the maximum overtaking revenue is decreasing in p_1, it follows from the second claim that the maximum revenue as a duopolist for any $p_1 < \overline{p}$ is smaller than the maximum overtaking revenue at the same p_1.

The derivation of $\overline{p}$ and the proof of the two claims depend on whether the price pair $(\overline{p}, \lambda^{-1}(\overline{p}))$ is located at the boundary between D_{21} and S_2 where $c_2 > a$ or $c_2 = a$. We will assume $c_2 > a$; the other case is similar. Consider the problem of choosing c_2 to maximize the revenue for matchmaker 2 in D_{21}, given by $(F(c_1) - F(c_2))c_2\mu(c_2, c_1)$, where c_1 satisfies $p_1 = c_1(\mu(c_1, b) - \mu(c_2, c_1)) + c_2\mu(c_2, c_1)$. Under the uniform type distribution, the above relation becomes $p_1 = \frac{1}{2}(c_1 b + c_2^2)$, and so $\mathrm{d}c_1/\mathrm{d}c_2 = -2c_2/b$. One can verify that matchmaker 2's revenue is concave in c_2, and thus the optimal c_2 satisfies the following first order condition:

$$\frac{1}{2}c_1^2 - \left(\frac{2c_1}{b} + \frac{3}{2}\right)c_2^2 = 0. \tag{A.1}$$

Since $c_1 = b$ at the boundary between S_2 and D_{21}, there is a unique $\overline{p}_1 = \frac{4}{7}b^2$ such that (A.1) holds with equality at $p_2 = \lambda^{-1}(\overline{p}_1)$. Further, straightforward calculations reveal that at $\overline{p}_1 = \frac{4}{7}b^2$, matchmaker 2's maximum undercutting revenue is smaller than its maximum overtaking revenue. Thus, we have established both claims mentioned above.

A.5. Lemma A.2 and Proof

Lemma A.2. *Under the uniform type distribution, matchmaker 2's revenue function is concave in c_2 for any p_1 in the D_{12} region.*

Proof. There are three cases, depending on p_1. In the first case, we have $p_1 \leq a^2$, which implies $c_1 = a$. Under uniform type distribution, the derivative of (3.5) with respect to c_2 is proportional to $-p_1 + (b-a)(b-2c_2)/2$. Thus, matchmaker 2's revenue is concave in c_2. In the second case, we have $p_1 \geq a\mu(a,b)$, which implies $c_1 > a$ and given by $c_1\mu(c_1,c_2) = p_1$. Under uniform type distribution, using constraint (3.6) and differentiating (3.5) twice with respect to c_2, we find that the revenue function is concave in c_2, if

$$-\left(b - c_1 + \frac{c_1c_2}{2c_1 + c_2}\right) + \frac{(b-c_2)c_1}{2c_1+c_2} < 0.$$

This is equivalent to $-b(c_1 + c_2) + 2c_1^2 - c_1c_2 < 0$, which is true because $c_1 \leq c_2 \leq b$. In the third case, we have $p_1 \in (a^2, a\mu(a,b))$, and there is a critical value $\overline{c}_2$ satisfying $a\mu(a,\overline{c}_2) = p_1$ such that $c_1 > a$ for $c_2 < \overline{c}_2$ and $c_1 = a$ for $c_2 \geq \overline{c}_2$. By constraint (3.6), c_1 decreases in c_2 to the left of $\overline{c}_2$ and is constant to the right. It then follows from the revenue function (3.5) that the derivative with respect to c_2 jumps down at $\overline{c}_2$. Since the revenue function is concave to either side of the kink, it is globally concave in c_2.

A.6. Proof of Lemma 3.9

We only need to consider the case where the efficient c_1^* for the planner is interior. By differentiating the objective function (3.13) with respect to c_1, we find that the efficient thresholds c_1^* and c_2^* satisfy the first order condition $f(c_1^*)(\mu(c_1^*, c_2^*) - 2c_1^*) = 0$. It then follows from (3.9) that $\rho(c_1^*) > c_1^*$. Since $\rho(\hat{c}_1) \leq \hat{c}_1$ by Proposition 3.5, the lemma follows from Assumption 2.2.

A.7. Proof of Lemma 3.10

Using the identity

$$(F(c_2) - F(c_1))\mu(c_1,c_2) + (1 - F(c_2))\mu(c_2,b) = (1 - F(c_1))\mu(c_1,b), \qquad (A.2)$$

we can rewrite the objective function of the planner (3.13) as

$$(1 - F(c_1))(\mu^2(c_1, b) + (\mu(c_1, b) - \mu(c_1, c_2))(\mu(c_2, b) - \mu(c_1, b))),$$

and the objective function of the monopolist (3.2) as

$$(1 - F(c_1))(c_1\mu(c_1, b) + (\mu(c_1, b) - \mu(c_1, c_2))(c_2 - c_1)).$$

Note that for the monopolist for any c_1 adding a second market always increases its revenue. The first order condition with respect to c_2 is

$$\frac{\mu_r(c_1, c_2^*)}{\mu(c_1, b) - \mu(c_1, c_2^*)} = \frac{\mu_l(c_2^*, b)}{\mu(c_2^*, b) - \mu(c_1, b)} \tag{A.3}$$

for the planner's problem, and

$$\frac{\mu_r(c_1, \hat{c}_2)}{\mu(c_1, b) - \mu(c_1, \hat{c}_2)} = \frac{1}{\hat{c}_2 - c_1} \tag{A.4}$$

for the monopolist's problem. It follows from comparing (A.3) to (A.4) that $c_2^* = \hat{c}_2$ if

$$\mu_l(c_2, b) = \frac{\mu(c_2, b) - \mu(c_1, b)}{c_2 - c_1}$$

for any c_2. This holds if $\mu(\cdot, b)$ is linear, including the uniform type distribution.

In the proof of Proposition 3.8 we have already established that for any c_1 there is a unique $\hat{c}_2$ that satisfies the first order condition (A.4). It remains to argue that there is a unique interior solution in c_2^* to the planner's problem. Using equation (A.2), we can rewrite the first order condition (A.3) as

$$f(c_2^*)(\mu(c_2^*, b) - \mu(c_1, c_2^*))(\mu(c_1, c_2^*) + \mu(c_2^*, b) - 2c_2^*) = 0.$$

For any c_1, there exists at least one c_2^* that satisfies the above first order condition, as $\mu(c_1, c_1) + \mu(c_1, b) \geq 2c_1$ and $\mu(c_1, b) + \mu(b, b) \leq 2b$. Such c_2^* is unique too, because under Assumption 2.2 we have $\mu_r(c_1, c_2) + \mu_l(c_2, b) \leq \frac{1}{2} + 1 < 2$ for any c_2.

A.8. Lemma A.3 and Proof

LEMMA A.3. *Under the uniform type distribution, there is no equilibrium with a dual market structure when* $a/b \geq \frac{1}{2}$.

PROOF. Condition (3.8) is necessary for an equilibrium with D_{12} and a matchmaker 1's price $\tilde{p}_1 > a^2$. This is because for any such $\tilde{p}_1$, matchmaker 2 has the option of charging $p_2 = \theta(\tilde{p}_1)$ to overtake matchmaker 1, which leads to $c_1 > a$. Under uniform distribution (3.8) becomes $c_1 < (\sqrt{19} - 4)b$, and since it cannot be satisfied when $a/b \geq \frac{1}{2}$, there is no equilibrium with a dual market structure in which $\tilde{p}_1 > a^2$. Further, condition (3.1) is necessary for an equilibrium with $\tilde{p}_1 \leq a^2$; this is because for any such $\tilde{p}_1$, matchmaker 2 can overtake matchmaker 1 by charging $p_2 = \theta(\tilde{p}_1)$, which leads to $c_1 = a$. This condition is violated for any $\tilde{p}_1$ if $\mu(a, b) \leq \frac{3}{2}a$, or $a/b \geq \frac{1}{2}$ for the uniform distribution.

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